Indian J. Phys. **78 (6)**, 431–437 (2004)

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Studies on microwave lamellar reflection gratings

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Received 10 February 2004, accepted 20 April 2004

Abstract : We derive the expression for the intensity of the diffracted waves from a lamellar reflection grating. We attempt the problem by examining the effects of the elevations and that of the depressions separately and finally derive the resultant intensity.

If suitably designed, the applicability of such a grating may be extended to the microwave regions apart from the normal optical and near infra-red region in which it is mostly used. Such a microwave reflection lamellar grating can have significant applications in astronomy and astrophysics. By proper choice of geometrical parameters and modifying its construction with suitable materials, a lamellar grating may be used as a microwave antenna in space communications as well.

Keywords : Microwave diffraction, lamellar gratings, microwave antenna.

PACS No. : 84.40.Az, 84.40.Ba, 42.79.Dj, 84.40.Ua

1. Introduction

The use of grating-based instruments is gaining importance in optical spectroscopy in astronomical observations because of their ability to simultaneously record many spectral elements at very low noise-bandwidth. The use of gratings is found very useful in infra-red regions also [1–3]. We anticipate that when suitably designed, they could as well be used in microwave region. In fact, we have studied these properties and all possibilities in a series of papers [4–6].

Now a days, several optical principles are being used in the construction of microwave antenna [7–10]. Microwaves and millimeter waves are being used in remote sensing [3], various detectors [11], measuring reflectivity in crystals [12], satellite communications [13] *etc*. In this communication, we represent the theory, construction and prospective uses of reflection type microwave lamellar grating. A simple lamellar reflection grating consists of a rectangular laminar profile. Such gratings can concentrate most of the diffracted energy into the first order so that the spectrum is highly intense and at the same time quite pure [14]. Lamellar dielectric gratings function efficiently as wave guide [15,16].

2. Construction of microwave lamellar grating

To construct a simple lamellar grating for the millimeter and microwaves, a highly polished metal surface may be suitably shaped to have equispaced corrugated grooves of desired depth and fabricated over a quartz substrate as shown in Figure 1. Quartz is an excellent absorber of microwaves and will prevent transmission of these waves across the grating.

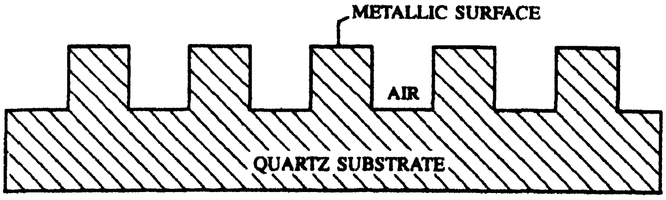

Figure 1. Schematic diagram of a simple lamellar grating.

Also, the metal coating over the quartz need not be very thick, because the skin depth of the microwaves is

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quite small compared to the wavelengths involved. For example at 10 GHz, the skin depth of copper is only 0.5 micron. The skin depths of some of the metals which can be used to construct lamellar grating are given in Figure 2 [16]. The utility of such gratings have been discussed elsewhere [17].

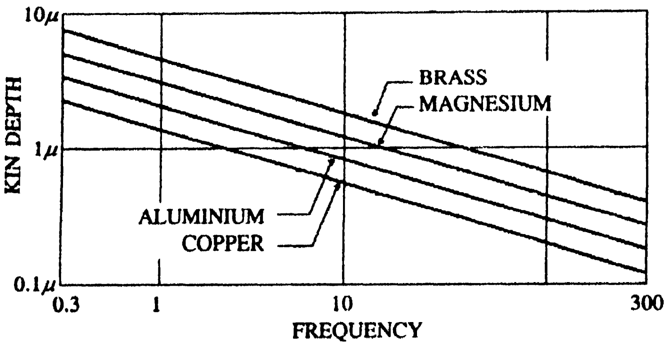

Figure 2. Diffracted waves from a lamellar grating.

3. Theory of reflection lamellar grating

For examining the diffracted waves from such a grating, let us consider Figure 3 where OA is the incident wave front and OB is the diffracted wave front, the angle of diffraction being θ. For our theoretical analysis, we have used the scalar wave diffraction theory and assumed normal incidence. Oblique incidence can be incorporated by including the angle of incidence in the diffraction equation; but for all practical purposes, it is desirable to have normal incidence, so that the loss due to surface plasmon waves on a metallic grating is minimum [18].

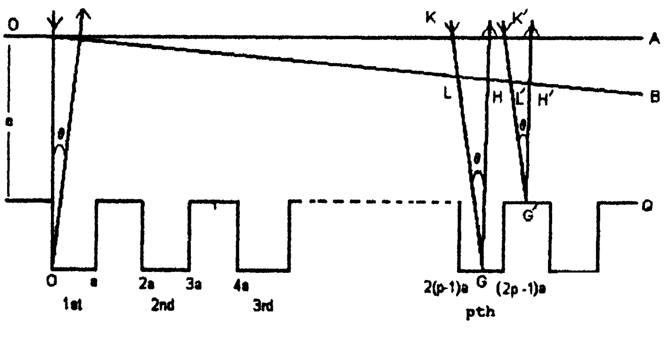

Figure 3. Skin depth of microwave in different metals.

Let the breadths of alternate elevations and depressions of the grating be each equal to 'a'. Let the depth of the grooves be 'd'. The value of the grating element is hence $(a + a)$. We suppose that the entire grating surface is divided into two parts :

(i) consisting of elevations only which we shall represent as E-grating,

(ii) consisting of depressions only which we shall represent as D-grating.

The D-type grating :

Let us designate the path difference between incident and diffracted wave fronts from a point G in the p-th depression by ΔpD.

$$\begin{aligned}\therefore \Delta pD &= \mathrm{KG} + \mathrm{GH}\\ &= \mathrm{KG} + (\mathrm{KG} - \mathrm{KL})\cos\theta\\ &= \mathrm{KG}\,(1 + \cos\theta) - \mathrm{OK}\sin\theta\\ &= (e + d)\,(1 + \cos\theta) - x\sin\theta\end{aligned}$$

where $\mathrm{OK} = x$.

$$= \alpha + \beta x,$$

where $\alpha = (e + d)\,(1 + \cos\theta)$

and $\beta = -\sin\theta$.

Let the equation of vibration on the incident wave front be $y_0 = \exp\{ikct\}$,

where, $k = \dfrac{2\pi}{\lambda}$, $i = \sqrt{-1}$ and c is the velocity of light.

Let all the diffracted waves at angle θ be brought to focus at F by a converging lens (not shown in the figure). The displacement at F due to Secondary Waves coming from a small element dx of the incident wave front at K is represented by

$$\begin{aligned}dS_{PD} &= \exp\{ik(ct - \Delta pD)\}dx\\ &= \exp\{ik(ct - \alpha)\}\exp(-ik\beta x)dx.\end{aligned}$$

The amplitude generated by the p-th element of depression at the focus F is given by

$$S_{PD} = \exp\{ik(ct-\alpha)\}\int_{2(p-1)a}^{(2p-1)a}\exp(-ik\beta x)\,dx$$

$$= \frac{1}{-ik\beta}\exp\{ik(ct-\alpha)\}\exp\{-2ik\beta(p-1)a\}$$

$$\times\left[\exp(-ik\beta a)-1\right]. \tag{1}$$

Let the total number of depressions be N. The total amplitude hence generated at the focus is given by

$$S_D = \frac{1}{-ik\beta}\{ik(ct-\alpha+2\beta a)\}\times\{\exp(-ik\beta a)-1\}$$

$$\times\left[\exp(-2ik\beta a)+\exp(-4ik\beta a)+\right.$$

$$\exp(-6ik\beta a)+\ldots\ldots\ldots+\exp(-2Nik\beta a)]$$

$$= \frac{1}{-ik\beta}\exp\{ik(ct-a)\}\{\exp(-ik\beta a)-1\}$$

$$\times\left[\frac{1-\exp(-2Nik\beta a)}{1-\exp(-2ik\beta a)}\right]. \quad (2)$$

Multiplying the complex amplitude by its conjugate and simplifying, we get the real amplitude D of vibration of this D-grating at the focus F given by

$$D = \frac{a}{\frac{k\beta a}{2}} \cdot \frac{\sin k\beta a}{2} \frac{\sin Nk\beta a}{\sin k\beta a} \quad (3)$$

Hence, the equation of vibration due to D-grating is given by

$$Y_D = D \cos k(ct - a), \quad (4)$$

where Y_D denotes the displacement of the ether particles.

E-type grating :

In this grating the path difference for the p-th elevation is given by

$$\Delta pE = K'G' + H'G'$$

$$= K'G'\ (1 + \cos\theta) - OK'\sin\theta.$$

If we denote $\eta = e(1 + \cos\theta)$ and $\beta = -\sin\theta$ we get,

$$\Delta pE = \eta + \beta x.$$

The displacement at the focus F due to an element dx of the incident wave front at a distance O is denoted by

$$dS_{PE} = \exp\ \{ik(ct - \eta)\}\exp(-ik\beta x)dx. \quad (5)$$

Hence the displacement due to the p-th element of the grating is given by

$$S_{PE} = \exp\{ik(ct-\eta)\}\int_{(2p-1)a}^{2pa}\exp(-ik\beta x)\,dx$$

$$= \frac{1}{-ik\beta}\exp\{ik\ (ct-\eta)\}\exp$$

$$(-2ik\beta pa)\times[1-\exp(ik\beta a)], \quad (6)$$

$$S_E = \frac{1}{-ik\beta}\exp\{ik(ct-\eta)\}\{1-\exp(ik\beta a)\}$$

$$\begin{bmatrix}\exp(-2ik\beta a)+\exp(-3ik\beta a)\\ +\exp(-6ik\beta a)+\ldots..+\exp(-2Nik\beta a)\end{bmatrix}$$

$$= \frac{1}{-ik\beta}.\exp\{ik(ct-\eta-2\beta a)\}$$

$$\{1-\exp(ik\beta a)\}\times\left[\frac{1-\exp(-2Nik\beta a)}{1-\exp(-2ik\beta a)}\right]. \quad (7)$$

Multiplying the complex amplitude by its conjugate and simplifying, we get the real amplitude E given by

$$E = \frac{a}{k\beta a}\sin\frac{k\beta a}{2}.\frac{\sin Nk\beta a}{\sin k\beta a} \quad (8)$$

The equation of vibration due to E-grating is therefore,

$$Y_E = \frac{a}{\frac{k\beta a}{2}}\,Sin\,\frac{k\beta a}{2}.\frac{\sin Nk\beta a}{\sin k\beta a}.\cos(ct-\eta-2\beta a). \quad (9)$$

The resultant intensity expression J is obtained by adding vectorically Y_D and Y_E.

Hence,

$$J = (D+E)^2 - 4ED\sin^2\frac{k}{2}(\delta_1-\delta_2) \quad (10)$$

where $\delta_1 = -\alpha$ and $\delta_2 = -\eta - 2\beta a.$

Substituting the values of δ_1 and δ_2 and $\alpha = (e + d)$ $(1 + \cos\theta)$, $\beta = -\sin\theta$ and $\eta = e(1 + \cos\theta)$ we have,

$$J = \frac{4a^2}{\left(\pi a\frac{\sin\theta}{x}\right)} \times\left[\sin^2\pi a\sin\theta\right]$$

$$\frac{\sin^2\left(\frac{2\pi Na\sin^2\theta}{\lambda}\right)}{\sin^2\left(\frac{2\pi a\sin\theta}{\lambda}\right)}$$

$$\times\cos^2\frac{\pi}{\lambda}\begin{bmatrix}d(1+\cos\theta)\\ +2a\sin\theta\end{bmatrix}. \quad (11)$$

The maximum of the primary spectrum *i.e.*, J becomes maximum when

$$\sin\frac{\pi a\sin\theta}{\lambda} = 1 = \sin\frac{s\pi}{2}$$

where

$s = 1, 2, 3,\ldots\ldots\ldots$

or $2a\sin\theta = s\lambda$

or $\sin\theta = \dfrac{s\lambda}{2a}$

or $\cos\theta\, d\theta = \dfrac{s}{2a}\, d\lambda$

$$\text{or} \quad \frac{d\theta}{d\lambda} = \frac{s}{2a\cos\theta} \tag{12}$$

The above expression gives the dispersive power. We find that the intensity

$J = 0$ occurs if $\sin^2\dfrac{2\pi Na\sin\theta}{\lambda} = 0 = \sin^2 s\pi$.

Hence, $\dfrac{2\pi Na\sin\theta}{\lambda} = s\pi$

$$\text{or} \quad \sin\theta = \frac{s\lambda}{2Na} \tag{13}$$

Also $J = 0$ if $\cos^2\dfrac{\pi}{\lambda}\{d(1+\cos\theta)+2a\sin\theta\} = 0$.

Putting $\psi = \dfrac{\pi}{\lambda}\{d\,(1+\cos\theta)+2a\sin\theta\}$, we have $\cos^2\psi$

$= 0 = \cos 2\,(2n+1)\dfrac{\pi}{2}$

or $2a\sin\theta = (2n+1)\dfrac{\lambda}{2} - d\,(1+\cos\theta)$.

Again, J has a maximum value when

$\sin^2\dfrac{2\pi a\sin\theta}{\lambda} = 0 = \sin^2 s\pi$

or $\dfrac{2\pi a\sin\theta}{\lambda} = s\pi$

$$\text{or} \quad \sin\theta = \frac{s\lambda}{2a} \tag{14}$$

Putting $\sin\theta = p$, the separation between the primary maximum of order S and neighbouring minimum given by eq. (13) is

$$\Delta p = \frac{\lambda}{2Na}. \tag{15}$$

If the wavelength is changed by $\Delta\lambda$, the s-th order principal maximum is displaced by

$$\Delta p' = \frac{s\Delta\lambda}{2a}. \tag{16}$$

On the limit of resolution in the s-th order,

$$\frac{\lambda}{2Na} = s.\frac{\Delta\lambda}{2a}$$

$$\text{or} \quad \frac{\lambda}{\Delta\lambda} = s.N. \tag{17}$$

The above equation gives the expression for the resolving power. It is obvious that the resolving power is given by the product of order number of the spectrum and the total number of elevations or depressions.

4. Results and discussion

Eq. (11) provides the expression for intensity of the beam diffracted by a lamellar grating as a function of various geometrical parameters of the grating and the wavelength of the incident beam. The plot of J vs λ is shown in Figure 4. The parameters chosen are $a = 1$ cm, $d = 1$, $N = 40$ and $\theta = 15°$.

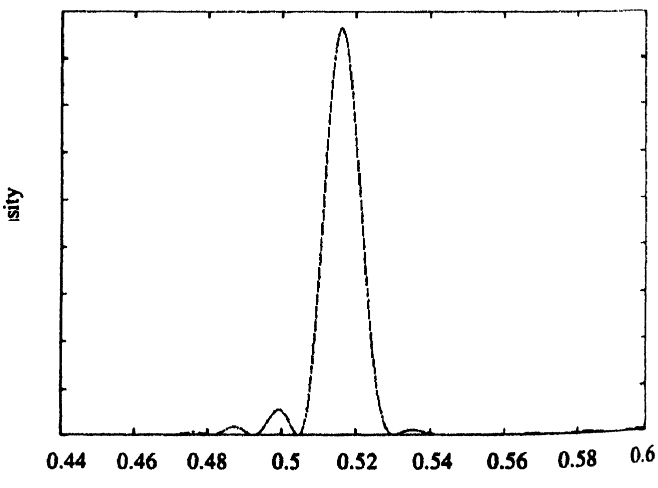

Figure 4. Intensity distribution (J) as a function of the wavelength λ for microwave reflection lamellar grating in the mm wave region. Parameters are $a = d = 1$ cm, $N = 40$ and $\theta = 15°$. Note the strong peak at $\lambda = 0.52$ cm.

As seen from the graph, the curve exhibits a well defined sharp peak at 0.52 cm. The graph shows that the intensity is highly wavelength sensitive. Changing the value to $N = 50$, the peak remains still at 0.52 cm. Reducing the value of N to 30 or less, the intensity pattern becomes worse and peak diminishes and hence the figure is not shown. The variation of J with angle of diffraction for a particular wavelength is shown in Figure 5. The geometrical parameters chosen are $a = d = 1$ cm, $N = 40$ and $\lambda = 0.5$ cm.

The principal maximum of zeroth order is at $\theta = 0°$. The principal maximum of first order is the second smaller peak at 14° with diminished intensity. Evidently, most of the light energy is confined within very short range (2.5°).

The expression for J can be used for designing microwave lamellar grating for the required wavelength. For example, to find out the most suitable grating parameters a and d with $N = 40$ at $\lambda = 0.5$ cm, we draw the 3-dimensional plot of intensity J (z-axis) when 'a' and 'd' change from 1 cm to 5 cm along x-and y-axis respectively (Figure 6).

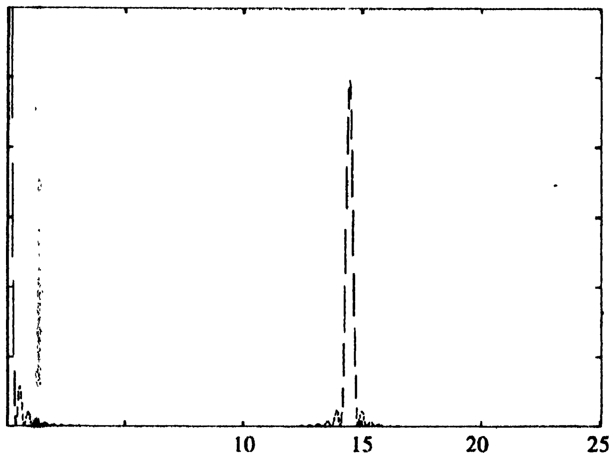

Figure 5. Intensity distribution (J) as a function of angle of diffraction θ. Parameters are $a = d = 1$ cm, $N = 40$, $\lambda = 0.5$ cm.

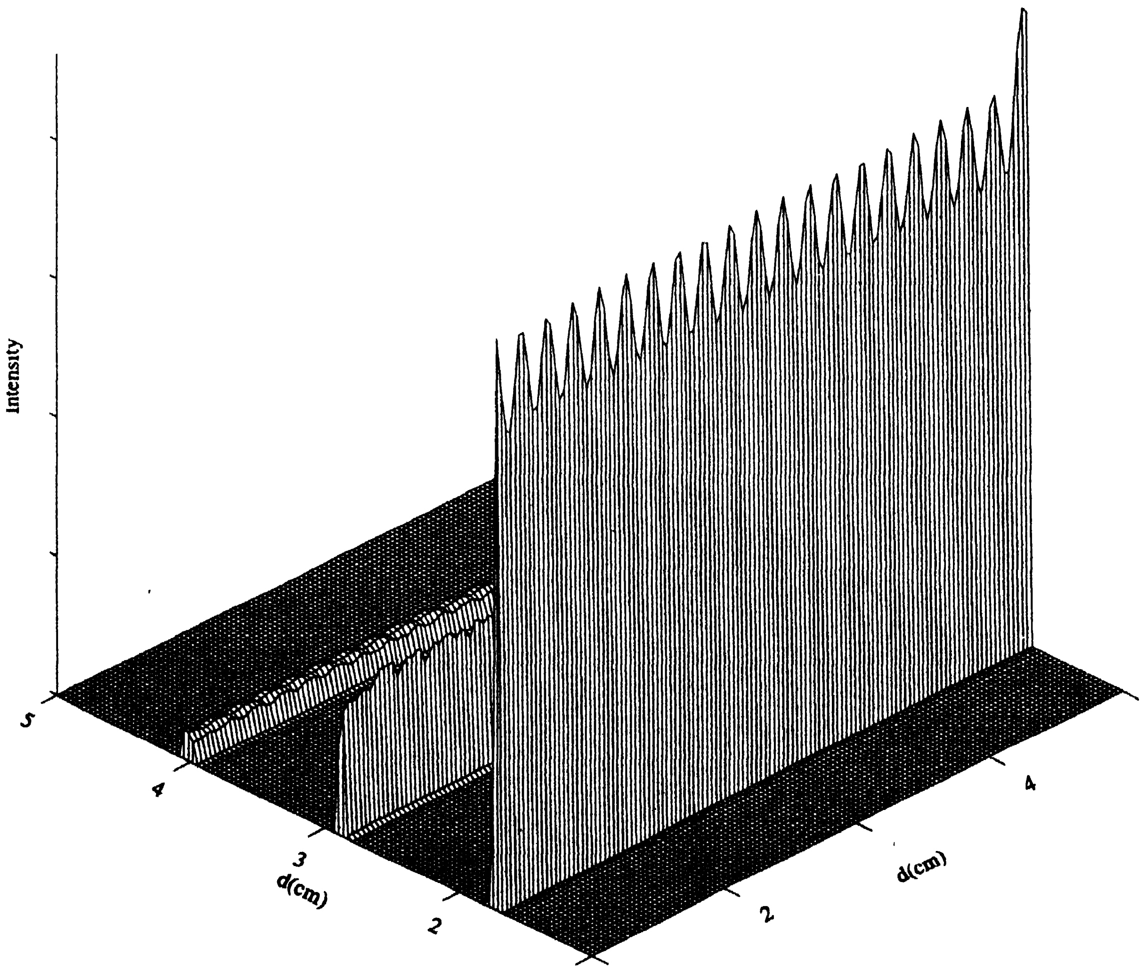

Figure 6. Intensity distribution (J) as a function of the geometrical parameters a and d of the grating at 0.5 cm.

We note that the minimum values of a and d sufficient for optimum intensity response for wavelength $\lambda = 0.5$ cm is $a \sim 1$ cm and $d \sim 2.5$ cm when $N = 40$. Knowledge of the minimum values of the parameters for optimum response will reduce the construction cost and is important from the economic point of view.

If the above grating is to be used for different wavelengths, study of the 3-dimensional plot of J as a function of θ and λ is necessary. Thus the position of the detector to get best response for the required wavelength can be selected from the graph. A 3-dimensional plot of J (z-axis) with $a = 1$ cm and $d = 2.5$ cm, $N = 40$ is shown in Figure 7. The wavelength changes from 1 mm to 10 mm approximately along x-axis and θ changes from 5° to ~15° along the y-axis.

Acknowledgments

We wish to express our sincere thanks to Dr. K K Dey, Ex-Professor of Physics, Banaras Hindu University and Prof. A K Sen of Institute of Radio Physics and Electronics, Calcutta University for valuable suggestions. S Chatterjee and U. Chattopadhyay thank Dr. A Chatterjee of Malda College for discussions. This work is partly supported by ISRO sponsored RESPOND project on Synthesis of bio-molecules during star formation and their detection with millimeter and microwave gratings.

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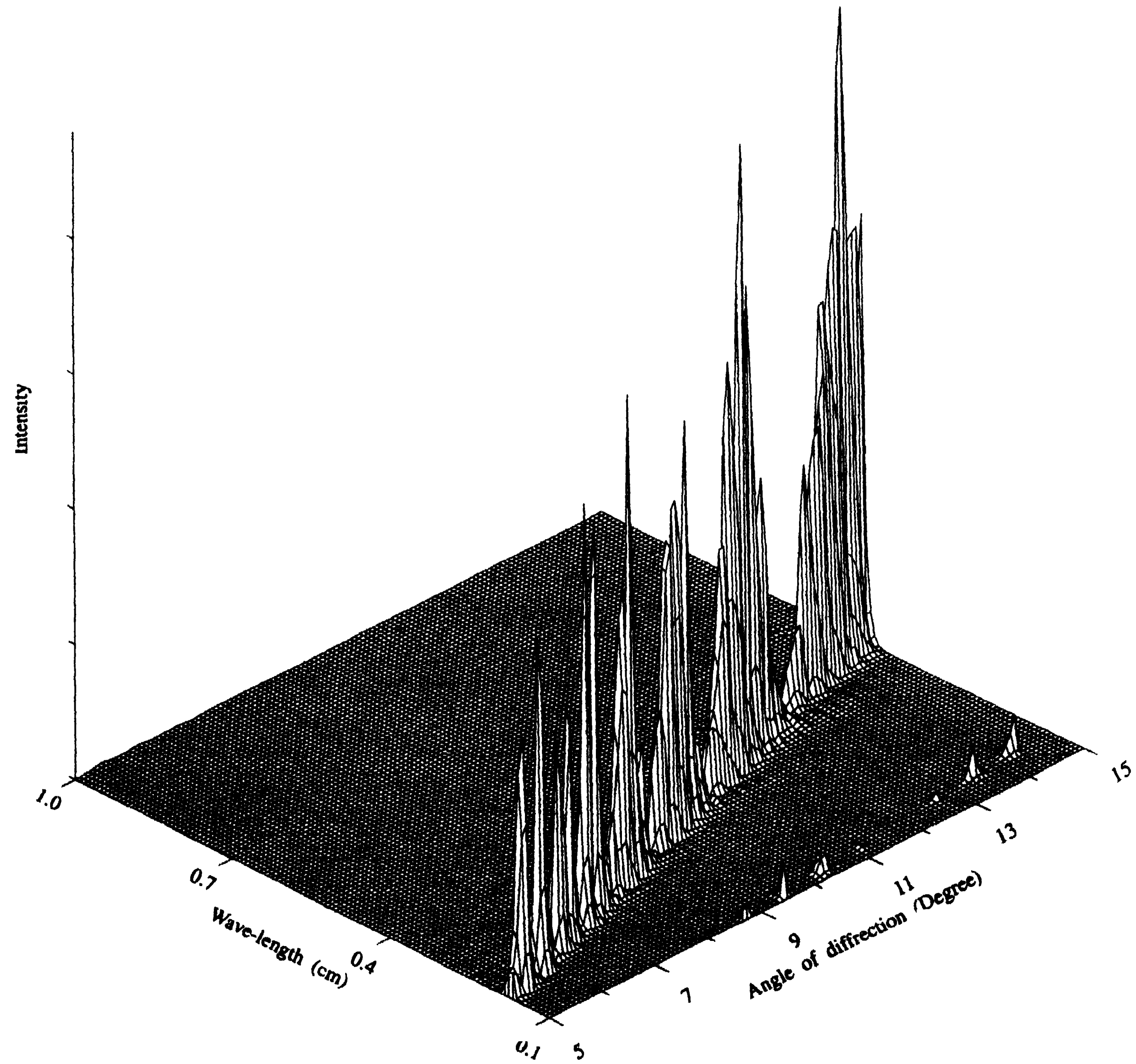

Figure 7. Intensity distribution (J) as a function of wavelength λ and diffraction angle θ

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